

# Exact non-Markovian cavity dynamics strongly coupled to a reservoir

Heng-Na Xiong,<sup>1,2</sup> Wei-Min Zhang,<sup>1,\*</sup> Xiaoguang Wang,<sup>2</sup> and Meng-Hsiu Wu<sup>1</sup>

<sup>1</sup>*Department of Physics and Center for Quantum Information Science,  
National Cheng Kung University, Tainan 70101, Taiwan*

<sup>2</sup>*Zhejiang Institute of Modern Physics and Department of Physics,  
Zhejiang University, Hangzhou, People's Republic of China*

(Dated: May 6, 2010)

The exact non-Markovian dynamics of a microcavity strongly coupled to a general reservoir at arbitrary temperature is studied. With the exact master equation for the reduced density operator of the cavity system, we analytically solve the time evolution of the cavity state and the associated physical observables. We show that the non-Markovian dynamics is completely determined by the propagating (retarded) and correlation Green functions. Compare the non-Markovian behavior at finite temperature with those at zero-temperature limit or Born-Markov limit, we find that the non-Markovian memory effect can dramatically change the coherent and thermal dynamics of the cavity. We also numerically study the dissipation dynamics of the cavity through the mean mode amplitude decay and the average photon number decay in the microwave regime. It is shown that the strong coupling between the cavity and the reservoir results in a long-time dissipationless evolution to the cavity field amplitude, and its noise dynamics undergoes a critical transition from the weak to strong coupling due to the non-Markovian memory effect.

PACS numbers: 42.50.-p, 03.65.Yz, 42.79.Gn

## I. INTRODUCTION

The dissipation quantum dynamics of optical cavities has been well investigated and deeply understood under the Born-Markov (BM) approximation [1]. The BM approximation is valid when the coupling between the system and the environment is weak enough so that the perturbation is applied, and meantime the characteristic time of the environment is sufficiently shorter than that of the system so that the non-Markovian memory effect is negligible. However, in many situations in the recent development of optical microcavities, the strong coupling effect or the long-time memory effect has become an important factor in controlling cavity dynamics. Typical examples include optical fields propagating in cavity arrays or in an optical fiber [2–4], trapped ions subjected to artificial colored noise [5–8], and microcavities interacting with a coupled resonator optical waveguide (CROW) or photonic crystals [9–16], etc. Specifically, for the trapped ions coupled with an engineered reservoir, the change of the characteristic frequency of the reservoir can be accomplished simply by applying a random electric field through a band-pass filter defining the frequency spectrum of the reservoir [5]. While, for a cavity interacting with CROW or photonic crystals, the coupling between them is controllable by changing the geometrical parameters of the defect cavity and the distance between the cavity and the CROW [15]. Both of them provide non-Markovian dissipation and decoherence channels [6–8, 16]. These strong coupling or long-time memory effects result in a complicated non-

Markovian process in cavity systems that has become a crucial concern for the rapid development of quantum information and quantum computation in terms of photons [17]. The non-Markovian behavior of the trapped ions has been discussed in many works [6–8]. In this paper, we shall investigate the non-Markovian dynamics for the second case, the cavity strongly coupled with its environment (CROW or photonic crystals).

Quantum dynamics of cavity systems is completely described by the master equation of the reduced density operator by taking the cavity as an open system. The master equation under the BM approximation can be found in many textbooks [1, 18, 19]. However, an exact master equation beyond the BM approximation is only derived in a few works. The first exact master equation, also called as Hu-Paz-Zhang master equation for quantum Brownian motion, was found almost two decades ago [20], in terms of Wigner distribution function in phase space via Feynman-Vernon influence functional [21]. Recently, our group developed exact master equations of the reduced density operator for both the fermion systems [22, 23] and the bosonic systems [24, 25], by using an extended Feynman-Vernon influence functional. These exact master equations can fully depict the quantum dissipative and decoherence dynamics in various open systems in the strong coupling regime. For the cavity system, the master equation obtained in [24, 25] is for zero-temperature. Here we shall extend it to a finite temperature. In practical, the temperature effect is also unavoidable and non-negligible. It has been pointed out [1] that for cavity frequency lies in the microwave regime, thermal photons are presented even at liquid helium temperatures [26]. In this paper, we shall use the exact master equation which is valid at arbitrary temperature to investigate the exact non-Markovian dynamics of a general cavity

---

\*Electronic address: wzhang@mail.ncku.edu.tw

strongly coupled with its reservoir, and to find general features of the coupling and temperature dependence in non-Markovian dynamics.

The paper is organized as follows. In Sec. II, we introduce the exact master equation we developed recently for the reduced density operator of the cavity system coupled to a general reservoir, from which the second-order perturbation master equation and the BM maser equation are reproduced at well-defined limits. In Sec. III, we then solve analytically the exact master equation via the coherent-state representation, included the exact solutions of mean field amplitude and the average photon number inside the cavity that experimentally measurable. We also obtain explicitly the reduced density matrix for three different initial states: the vacuum state, the coherent state and the mixed state, to show the different non-Markovian behaviors. The decoherence dynamics of cavity field is explicitly analyzed. In Sec. IV, we numerically demonstrate the exact non-Markovian behavior through the time-dependence of the mean mode amplitude and the average photon number in the cavity in both the weak and strong coupling regimes for three typical spectral densities, the Ohmic, sub-Ohmic and super-Ohmic cases, with the cavity's frequency being focused in the microwave regime. We find that the strong coupling between the cavity and the reservoir results in a long-time dissipationless evolution to the cavity field amplitude, and the noise dynamics undergoes a critical transition from the weak to strong coupling due to the non-Markovian memory effect. Finally a conclusion is given in Sec. V.

## II. EXACT MASTER EQUATION FOR A CAVITY IN A GENERAL RESERVOIR

We consider a cavity with a single mode coupled to a general reservoir. The Hamiltonian of the total system is given by

$$H = \hbar\omega_0 a^\dagger a + \sum_k \hbar\omega_k b_k^\dagger b_k + \sum_k \hbar V_k \left( b_k^\dagger a + a^\dagger b_k \right), \quad (1)$$

in which the first term is for the single cavity mode with  $a^\dagger, a$  being the creation and annihilation operators of the cavity field,  $\omega_0$  is its frequency; the second term is the Hamiltonian  $H_R$  for a general reservoir modeled as a collection of infinite harmonic oscillators, where  $b_k^\dagger$  and  $b_k$  are the corresponding creation and annihilation operators of the  $k$ th oscillator with the frequency  $\omega_k$ . The coupling between the cavity and the environment is described by the third term and  $V_k$  is the coupling strength between them, which is tunable for the reservoir being CROW or photonic crystals [9–16].

### A. Exact master equation

The exact master equation for the cavity field is given in terms of the reduced density operator which is defined from the density operator of the total system by tracing over entirely the environmental degrees of freedom:  $\rho(t) \equiv \text{tr}_R \rho_{\text{tot}}(t)$ , where the total density operator is governed by the quantum Liouville equation  $\dot{\rho}_{\text{tot}}(t) = e^{-\frac{i}{\hbar}H(t-t_0)} \rho_{\text{tot}}(t_0) e^{\frac{i}{\hbar}H(t-t_0)}$ . As usual [27], assuming that the cavity field is uncorrelated with the reservoir before the initial time  $t_0$ :  $\rho_{\text{tot}}(t_0) = \rho(t_0) \otimes \rho_R(t_0)$ , and the reservoir is initially in the equilibrium state:  $\rho_R(t_0) = \frac{1}{Z} e^{-H_R/(k_B T)}$  where  $k_B$  is the Boltzmann constant and  $T$  the reservoir's initial temperature. Then tracing over all the environmental degrees of freedom can be easily carried out using the Feynman-Vernon influence functional approach [21] in the framework of coherent state path-integral representation [28]. The resulting master equation for the reduced density operator has a standard form similar to the master equation for electrons in nanostructure we developed recently [22, 25] (with some sign difference due to the different statistical property between fermions and bosons):

$$\begin{aligned} \dot{\rho}(t) = & -i\omega'_0(t) [a^\dagger a, \rho(t)] \\ & + \kappa(t) \{2a\rho(t)a^\dagger - a^\dagger a\rho(t) - \rho(t)a^\dagger a\} \\ & + \tilde{\kappa}(t) \{a^\dagger \rho(t)a + a\rho(t)a^\dagger - a^\dagger a\rho(t) - \rho(t)aa^\dagger\}, \end{aligned} \quad (2)$$

where the time-dependent coefficient  $\omega'_0(t)$  is the renormalized frequency of the cavity, while  $\kappa(t)$  and  $\tilde{\kappa}(t)$  describe the dissipation and noise to the cavity field due to the coupling with the reservoir. These coefficients are non-perturbatively determined by the following relations:

$$\omega'_0(t) = -\text{Im}[\dot{u}(t)u^{-1}(t)], \quad (3a)$$

$$\kappa(t) = -\text{Re}[\dot{u}(t)u^{-1}(t)], \quad (3b)$$

$$\tilde{\kappa}(t) = \dot{v}(t) - 2v(t)\text{Re}[\dot{u}(t)u^{-1}(t)], \quad (3c)$$

and  $u(t)$  and  $v(t)$  satisfies the integrodifferential equations of motion:

$$\dot{u}(\tau) + i\omega_0 u(\tau) + \int_{t_0}^{\tau} d\tau' g(\tau - \tau') u(\tau') = 0, \quad (4a)$$

$$\begin{aligned} \dot{v}(\tau) + i\omega_0 v(\tau) + \int_{t_0}^{\tau} d\tau' g(\tau - \tau') v(\tau') \\ = \int_{t_0}^{\tau} d\tau' \tilde{g}(\tau - \tau') \bar{u}^*(\tau'), \end{aligned} \quad (4b)$$

subjected to the initial condition  $u(t_0) = 1$ , and  $\bar{u}(\tau) \equiv u(t + t_0 - \tau)$ . Note that the integral kernels in the above equations involve non-perturbatively the time correlation functions of the reservoirs:  $g(\tau - \tau')$  and  $\tilde{g}(\tau - \tau')$ . These two time-correlation functions characterize all the non-Markovian memory structures between the cavity and the reservoir. By defining the spectral density of

the reservoir  $J(\omega) = 2\pi \sum_k |V_k|^2 \delta(\omega - \omega_k)$ , the time-correlation functions can be expressed as

$$g(\tau - \tau') = \int_0^\infty \frac{d\omega}{2\pi} J(\omega) e^{-i\omega(\tau - \tau')}, \quad (5a)$$

$$\tilde{g}(\tau - \tau') = \int_0^\infty \frac{d\omega}{2\pi} J(\omega) \bar{n}(\omega, T) e^{-i\omega(\tau - \tau')}, \quad (5b)$$

where  $\bar{n}(\omega, T) = \frac{1}{e^{\hbar\omega/k_B T} - 1}$  is the average number distribution of the reservoir thermal excitation at the initial time  $t_0$ . If the reservoir spectrum is continuous,  $V_k \rightarrow V(\omega)$ , we have  $J(\omega) = 2\pi g(\omega) |V(\omega)|^2$  where  $g(\omega)$  is the density of state for the reservoir.

The master equation (2) is exact, far beyond the BM approximation widely used in quantum optics. The back-reaction effect between the system and environment is fully taken into account by the time-dependent coefficients,  $\omega'_0(t)$ ,  $\kappa(t)$  and  $\tilde{\kappa}(t)$ , in the master equation (2) through the integrodifferential equations (4). In fact, we can directly solve from Eq. (4)

$$\dot{u}(t)u^{-1}(t) = -i\omega_0 - \int_{t_0}^t d\tau g(t - \tau)u(\tau)u^{-1}(t), \quad (6a)$$

$$v(t) = \int_{t_0}^t d\tau_1 \int_{t_0}^{\tau_1} d\tau_2 \bar{u}(\tau_1) \tilde{g}(\tau_1 - \tau_2) \bar{u}^*(\tau_2). \quad (6b)$$

Then these time-dependent coefficients in the exact master equation can be simplified as

$$\omega'_0(t) = \omega_0 + \text{Im}[w(t)], \quad \kappa(t) = \text{Re}[w(t)], \quad (7a)$$

$$\tilde{\kappa}(t) = \dot{v}(t) + 2v(t)\text{Re}[w(t)]. \quad (7b)$$

with  $w(t) = \int_{t_0}^t d\tau g(t - \tau)u(\tau)u^{-1}(t)$ , which can be calculated by solving  $u(\tau)$  non-perturbatively from Eq. (4a). Thus, the non-Markovian memory structure is non-perturbatively built into the integral kernels in these equations. The expression of the integrodifferential equation (4) shows that  $u(t)$  is just the propagating function of the cavity field (the retarded Green function in nonequilibrium Green function theory [29]), and  $v(t)$  is the corresponding correlation (Green) function which is also determined by  $u(\tau)$  [see the equation (6b)]. Therefore, the exact master equation for cavity reduced density operator depicts the full nonequilibrium dynamics of the cavity system.

In fact, the exact master equation presented here simply covers the exact master equation at zero-temperature we derived very recently [24, 25]. Taking the zero-temperature limit  $T = 0$ , then  $\bar{n}(\omega, T) = 0$  so that  $\tilde{g}(\tau - \tau') = 0$ . As a result, we have  $v(t) = 0$  and therefore the coefficient  $\tilde{\kappa}(t) = 0$ . The master equation is simply reduced to

$$\begin{aligned} \dot{\rho}(t) = & -i\omega'_0(t) [a^\dagger a, \rho(t)] \\ & + \kappa(t) \{2a\rho(t)a^\dagger - a^\dagger a\rho(t) - \rho(t)a^\dagger a\}, \end{aligned} \quad (8)$$

with the temperature-independent coefficients  $\omega'_0(t)$ ,  $\kappa(t)$  obeying the same equation (3a-3b) through Eq. (4a).

Eq. (8) is the exact master equation for cavity fields coupled to the vacuum fluctuation (i.e. the zero temperature limit) [24, 25].

## B. Reduce to BM limit

Interestingly, the exact master equation (2) has the same form as the BM master equation in the literature [1]. The difference is the coefficients in the master equation. Here all the coefficients are time-dependent and determined by integrodifferential equations of motion (4) or (6), which non-perturbatively takes into account the back-reaction effects of the reservoir on the cavity field. While the coefficients in the BM master equation are all time-independent that ignores all the memory effects between the cavity and the reservoir and are determined under the perturbation approximation up to the second order of the coupling  $V(\omega)$  and then taking the Markov limit.

Explicitly, since the time correlation functions  $g(\tau - \tau')$  and  $\tilde{g}(\tau - \tau')$  are already proportional to  $|V(\omega)|^2$ , taking approximately the time-dependent coefficients in Eq. (7) up to the second-order of the coupling  $V(\omega)$  means that the propagating functions  $u(\tau)$  and  $u^{-1}(t)$  in the right hand side of Eq. (6) should be approximated only up to the zero-order:  $u_0(\tau) = e^{-i\omega_0(\tau - t_0)}$  and  $u_0^{-1}(t) = e^{i\omega_0(t - t_0)}$ . This leads to

$$\dot{u}(t)u^{-1}(t) \simeq -i\omega_0 - \int_{t_0}^t d\tau \int_0^\infty \frac{d\omega}{2\pi} J(\omega) e^{-i(\omega - \omega_0)(t - \tau)}, \quad (9a)$$

$$v(t) \simeq 2 \int_{t_0}^t d\tau \int_0^\infty \frac{d\omega}{2\pi} J(\omega) \bar{n}(\omega, T) \cos[(\omega - \omega_0)(t - \tau)]. \quad (9b)$$

The term  $2v(t)\text{Re}[w(t)]$  in (7b) is proportional to  $|V(\omega)|^4$  and should be ignored in the same approximation. Then the coefficients of Eq. (3) or (7) are reduced to

$$\omega'_0(t) \simeq \omega_0 - \int_{t_0}^t d\tau \int_0^\infty \frac{d\omega}{2\pi} J(\omega) \sin[(\omega - \omega_0)(t - \tau)], \quad (10a)$$

$$\kappa(t) \simeq \int_{t_0}^t d\tau \int_0^\infty \frac{d\omega}{2\pi} J(\omega) \cos[(\omega - \omega_0)(t - \tau)], \quad (10b)$$

$$\tilde{\kappa}(t) \simeq 2 \int_{t_0}^t d\tau \int_0^\infty \frac{d\omega}{2\pi} J(\omega) \bar{n}(\omega, T) \cos[(\omega - \omega_0)(t - \tau)]. \quad (10c)$$

Substituting these coefficients into Eq. (2) results in the master equation of the cavity field in the perturbation approximation up to the second order of the coupling constant between the cavity and the reservoir. This perturbative master equation is a good approximation only for the dissipation and noise dynamics of the cavity mode in the weak coupling regime.

To reproduce the standard BM master equation with the time-independent coefficients, one needs further to take the Markov limit where  $t$  is the typical time scale for the dynamics of the system and the  $t'$  integration is dominated by much shorter time characterizing the decay of reservoir correlations [1, 25]. In other words, one can take  $t'$  integration to infinity in the equation (10),

$$\lim_{t \rightarrow \infty} \int_0^{t-t_0} dt' e^{\pm i(\omega - \omega_0)t'} = \pi \delta(\omega - \omega_0) \mp i \frac{\mathcal{P}}{\omega - \omega_0}, \quad (11)$$

where  $\mathcal{P}$  denotes the principle value of the integral. As a result, all the coefficients in the master equation, given by (10), become time-independent:

$$\omega'_0 = \omega_0 + \delta\omega_0, \quad \kappa = \pi g(\omega_0) |V(\omega_0)|^2 = J(\omega_0)/2, \quad (12a)$$

$$\tilde{\kappa} = 2\pi g(\omega_0) |V(\omega_0)|^2 \bar{n}(\omega_0, T) = 2\kappa \bar{n}(\omega_0, T), \quad (12b)$$

and the frequency shift  $\delta\omega_0 = \mathcal{P} \int_0^\infty d\omega \frac{g(\omega) |V(\omega)|^2}{\omega - \omega_0}$ . Then the exact master equation (2) is reduced to

$$\begin{aligned} \dot{\rho}(t) = & -\frac{i}{\hbar}(\omega_0 + \delta\omega_0)[a^\dagger a, \rho(t)] \\ & + \kappa[2a\rho(t)a^\dagger - a^\dagger a\rho(t) - \rho(t)a^\dagger a] \\ & + 2\kappa\bar{n}(\omega_0, T)[a^\dagger \rho(t)a + a\rho(t)a^\dagger - a^\dagger a\rho(t) - \rho(t)aa^\dagger]. \end{aligned} \quad (13)$$

Eq. (12) with (13) reproduces exactly the BM master equation in quantum optics for a single cavity mode interacting with a thermal field [1].

### III. EXACT SOLUTION OF THE MASTER EQUATION

In this section, we shall present the exact solution of master equation for some physical observables and also for the reduced density operator of the cavity.

#### A. Exact solution for some physical observables

The main physical observables for a cavity are the decays of the mean mode amplitude and the average photon number inside the cavity. The mean mode amplitude of the cavity field is defined by  $\langle a(t) \rangle = \text{tr}[a\rho(t)]$ . From the exact master equation (2), it is easy to find that  $\langle a(t) \rangle$  obeys the equation of motion

$$\langle \dot{a}(t) \rangle = -[i\omega'_0(t) + \kappa(t)]\langle a(t) \rangle = \frac{\dot{u}(t)}{u(t)}\langle a(t) \rangle. \quad (14)$$

which has the exact solution:

$$\langle a(t) \rangle = u(t)\langle a(t_0) \rangle. \quad (15)$$

That is, the time evolution and the decay behavior of the mean mode amplitude is totally determined by  $u(t)$ .

Eq. (15) clearly indicates that  $u(t)$  is the propagating function characterizing the time evolution of the cavity field.

Another important physical observable is the average particle number inside the cavity, which is defined by  $n(t) = \text{tr}[a^\dagger a\rho(t)]$ . From the exact master equation, it is also easy to find that

$$\dot{n}(t) = -2\kappa(t)n(t) + \tilde{\kappa}(t). \quad (16)$$

On the other hand, Eq. (3b) can be rewritten as

$$\dot{v}(t) = -2\kappa(t)v(t) + \tilde{\kappa}(t), \quad (17)$$

with  $-2\kappa(t) = [\dot{u}/u(t) + \text{H.c.}]$ . Combing these equations together, we obtain the exact solution of  $n(t)$  in terms of  $u(t)$  and  $v(t)$ :

$$n(t) = u(t)n(t_0)u^*(t) + v(t). \quad (18)$$

This relationship is similar to the fermion case we derived recently [30]. In fact, the above solution is a result of the correlated Green function in nonequilibrium Green function theory [29]. Since  $v(t)$  is also determined by  $u(t)$  as we can see from Eq. (6b), both the mean mode amplitude and the average photon number inside the cavity are completely solved by the propagating function  $u(t)$ .

If we take the BM limit in which all the coefficients in the master equation are reduced to  $\omega'_0 = \omega_0 + \delta\omega_0, \kappa = J(\omega_0)/2, \tilde{\kappa} = 2\kappa\bar{n}(\omega_0, T)$  with  $\delta\omega_0 = P \int_0^\infty \frac{d\omega}{2\pi} \frac{J(\omega)}{\omega - \omega_0}$  [see Eq.(12)], then the resolution of  $u(t)$  simply becomes

$$u_{\text{BM}}(t) = e^{-(i\omega'_0 + \kappa)(t-t_0)}. \quad (19)$$

Correspondingly, the evolution of the mean mode amplitude in the BM limit is

$$\langle a(t) \rangle_{\text{BM}} = e^{-(i\omega'_0 + \kappa)(t-t_0)} \langle a(t_0) \rangle, \quad (20)$$

which shows an exponential decay in time. Substituting (19) into (4b), we have the BM solution of  $v(t)$ :

$$v_{\text{BM}}(t) = \bar{n}(\omega_0, T) [1 - e^{-2\kappa(t-t_0)}]. \quad (21)$$

Thus the average photon number in the BM limit is simply given by,

$$n_{\text{BM}}(t) = n(t_0)e^{-2\kappa(t-t_0)} + \bar{n}(\omega_0, T) [1 - e^{-2\kappa(t-t_0)}]. \quad (22)$$

These results reproduce all the BM solutions in weak coupling regime [1]. However, the exact solutions of Eqs. (15) and (18) allow us to explore the non-Markovian dynamics of the cavity systems not only in the weak coupling regime but also in the strong coupling regime. The explicit difference between the non-Markovian and Markov dynamics can be seen by comparing the solution of Eqs. (15) and (18) with (20) and (22).

TABLE I: Coefficients in the exact solution of the propagating function

	$A(t)$	$B(t)$	$C(t)$	$D(t)$
Exact result	$\frac{1}{1+v(t)}$	$\frac{u(t)}{1+v(t)}$	$\frac{v(t)}{1+v(t)}$	$1 - \frac{ u(t) ^2}{1+v(t)}$
$T \rightarrow 0$ limit	1	$u(t)$	0	$1 -  u(t) ^2$
BM limit*	$\frac{1}{1+v_{\text{BM}}(t)}$	$\frac{u_{\text{BM}}(t)}{1+v_{\text{BM}}(t)}$	$\frac{v_{\text{BM}}(t)}{1+v_{\text{BM}}(t)}$	$1 - \frac{ u_{\text{BM}}(t) ^2}{1+v_{\text{BM}}(t)}$

\*where  $u_{\text{BM}}(t)$  and  $v_{\text{BM}}(t)$  are given by Eqs. (19) and (21).

### B. Exact Solution of the reduced density operator

In fact, the reduced density operator can be also explicitly obtained through the coherent state representation. The reduced density matrix in the coherent state representation is given by

$$\rho(t) = \int d\mu(\alpha_f) d\mu(\alpha'_f) \rho(\alpha_f^*, \alpha'_f, t) |\alpha_f\rangle \langle \alpha'_f|, \quad (23)$$

where  $d\mu(\alpha) \equiv \frac{d\alpha^* d\alpha}{2\pi i} e^{-|\alpha|^2}$  is the integrate measure in the complex space of the coherent state  $|\alpha\rangle = e^{\alpha a^\dagger} |0\rangle$  [28], and  $\rho(\alpha_f^*, \alpha'_f, t) = \langle \alpha_f | \rho(t) | \alpha'_f \rangle$  can be obtained from the exact master equation:

$$\begin{aligned} \rho(\alpha_f^*, \alpha'_f, t) = & A(t) \int d\mu(\alpha_i) d\mu(\alpha'_i) \rho(\alpha_i^*, \alpha'_i, t_0) \\ & \times \exp \{ \alpha_f^* B(t) \alpha_i + \alpha_f^* C(t) \alpha'_i \\ & + \alpha_i^* D(t) \alpha_i + \alpha'_i{}^* B^*(t) \alpha'_f \}, \end{aligned} \quad (24)$$

where  $\rho(\alpha_i^*, \alpha'_i, t_0) = \langle \alpha_i | \rho(t_0) | \alpha'_i \rangle$  is the initial state  $\rho(t_0)$  in the coherent state representation. The exponential function in the integral is the propagating function of the reduced density operator, which fully determines the time evolution of the reduced density matrix in the coherent state representation. The time-dependent coefficients  $A(t)$ ,  $B(t)$ ,  $C(t)$  and  $D(t)$  in the propagating function are listed in table I for various cases. Thus, for a given initial state  $\rho(t_0)$ , the corresponding analytical solution of the reduced density operator for an arbitrary coupling to the reservoir at an arbitrary temperature can be obtained from Eqs. (23) and (24). Here we shall consider a few typical initial states: the vacuum state, the coherent state and the mixed state.

#### 1. Initial vacuum state

First, we consider the cavity initially in the vacuum state,

$$\rho(t_0) = |0\rangle \langle 0|. \quad (25)$$

From Eqs. (23) and (24), it is not difficult to obtain the exact reduced density operator at arbitrary time  $t$ ,

$$\begin{aligned} \rho(t) = & A(t) \sum_{n=0}^{\infty} [C(t)]^n |n\rangle \langle n| \\ = & \sum_{n=0}^{\infty} \frac{[v(t)]^n}{[1+v(t)]^{n+1}} |n\rangle \langle n|. \end{aligned} \quad (26)$$

This is a completely mixed state in terms of the Fock space of the cavity, with the average photon number

$$n(t) = v(t). \quad (27)$$

That is, there is no existence of a coherent field in the cavity if it is initially empty and also no external driving field is applied to the cavity. It indicates that the coupling to the reservoir makes the cavity continuously but randomly gain the energy (photons) from the reservoir, and eventually reach to a steady mixed state with a constant average photon number  $n(t) = v(t \rightarrow \infty)$ . However, we must point out that this steady limit is generally different from the BM limit since  $v(t \rightarrow \infty) \neq \bar{n}(\omega_0, T)$ . The difference is a manifestation of the non-Markovian effect that we shall demonstrate numerically in the next section.

At zero-temperature limit,  $v(t) = 0$ , then

$$\rho(t)|_{T=0} = |0\rangle \langle 0| = \rho(t_0). \quad (28)$$

In other words, the vacuum state remains unchanged if the reservoir temperature is zero. This is a trivial physical consequence since at zero temperature, the reservoir is also in the vacuum state. Then no photon can be exchanged between the cavity and the reservoir so that the cavity remains unchanged. On the other hand, in the BM limit, we have

$$\begin{aligned} \rho_{\text{BM}}(t) = & \sum_n \frac{[v_{\text{BM}}(t)]^n}{[1+v_{\text{BM}}(t)]^{n+1}} |n\rangle \langle n| \\ \stackrel{t \rightarrow \infty}{=} & \sum_n \frac{[\bar{n}(\omega_0, T)]^n}{[1+\bar{n}(\omega_0, T)]^{n+1}} |n\rangle \langle n|, \end{aligned} \quad (29)$$

which is a thermal equilibrium state with the average photon number  $n_{\text{BM}}(t) \rightarrow \bar{n}(\omega_0, T)$  at temperature  $T$ . It is expected that the solution of Eq. (26) can be very different from that of Eq. (29) in the strong coupling regime.

## 2. Initial coherent state

Next, we consider an initial coherent state,

$$\rho(t_0) = e^{-|\alpha_0|^2} |\alpha_0\rangle\langle\alpha_0|, \quad (30)$$

From Eqs. (23) and (24), we find that the reduced density operator at time  $t$  becomes

$$\begin{aligned} \rho(t) = \exp\left\{\frac{|\alpha(t)|^2}{1+v(t)}\right\} \sum_{n=0}^{\infty} \frac{[v(t)]^n}{[1+v(t)]^{n+1}} \\ \times \left| \frac{\alpha(t)}{1+v(t)}, n \right\rangle \left\langle \frac{\alpha(t)}{1+v(t)}, n \right|, \end{aligned} \quad (31)$$

where  $\left| \frac{\alpha(t)}{1+v(t)}, n \right\rangle \equiv \exp\left[\frac{\alpha(t)}{1+v(t)} a^\dagger\right] |n\rangle$  is defined as a generalized coherent state, and  $\alpha(t) = u(t)\alpha_0$ . It is interest to see that Eq. (31) is indeed a mixed state of generalized coherent states  $\left| \frac{\alpha(t)}{1+v(t)}, n \right\rangle$ . The average photon number in this state is given by

$$n(t) = |\alpha(t)|^2 + v(t), \quad (32)$$

as we expected.

In the weak coupling regime,  $u(t)$  generally decays to zero. The steady state limit of the above state will be the same as that in the case 1, and the asymptotic average photon number is also given by  $n(t) = v(t \rightarrow \infty)$ . The corresponding reduced density operator asymptotically becomes a completely mixed state of Eq. (26). This solution shows an exact decoherence process in the cavity. The decoherence arises from two sources, the mean mode amplitude damping characterized by the decay behavior in terms of the propagating function through the solution  $\alpha(t) = u(t)\alpha_0$ , and the thermal-fluctuation-induced noise effect characterized by  $v(t)$ , as we can see from Eq. (31). The later describes a process of randomly loss or gain thermal energy from the reservoir, up to the initial temperature of the reservoir.

However, in the strong coupling regime,  $u(t)$  may not decay to zero, as we shall show explicitly in the numerical calculation in the next section. Then the reduced density operator remains as a mixed coherent state. On the other hand, at zero-temperature limit ( $v(t) = 0$ ), the reduced density operator at time  $t$  is given by

$$\rho(t)|_{T=0} = e^{-|\alpha(t)|^2} |\alpha(t)\rangle\langle\alpha(t)|. \quad (33)$$

In other words, the cavity can remain in a coherent state in the zero temperature limit. These two features [ $u(t)$  may not decay to zero in the strong coupling regime and  $v(t) = 0$  at  $T = 0$ ] indicate that enhancing the coupling between the cavity and the reservoir and meantime lowering the initial temperature of the reservoir can significantly reduce the decoherence effect in cavity system.

On the other hand, in BM limit the reduced density operator is always reduced to a thermal state after a long time:

$$\rho_{\text{BM}}(t \rightarrow \infty) = \sum_{n=0}^{\infty} \frac{[\bar{n}(\omega_0, T)]^n}{[1 + \bar{n}(\omega_0, T)]^{n+1}} |n\rangle\langle n|, \quad (34)$$

This is because  $u_{\text{BM}}(t \rightarrow \infty)$  must approach to zero, see Eq. (19). It shows that the results in BM limit can be qualitatively different from the exact solution. These analytical results reveal the underlying mechanism of reservoir-induced decoherence in cavity dynamics, which may provide some insight how to control the decoherence dynamics in such systems.

## 3. Initial mixed state

The last special case we shall consider is the cavity in an initially mixed state,

$$\rho(t_0) = \sum_{n=0}^{\infty} \frac{[n(t_0)]^n}{[1 + n(t_0)]^{n+1}} |n\rangle\langle n|, \quad (35)$$

with the average initial photon number  $n(t_0)$ . Using the solution of the master equation, we have

$$\begin{aligned} \rho(t) = \frac{A(t)}{1 + n(t_0) - n(t_0)D(t)} \\ \times \sum_{n=0}^{\infty} \left( \frac{n(t_0)|B(t)|^2}{1 + n(t_0) - n(t_0)D(t)} + C(t) \right)^n |n\rangle\langle n| \\ = \sum_{n=0}^{\infty} \frac{[n(t)]^n}{[1 + n(t)]^{n+1}} |n\rangle\langle n|, \end{aligned} \quad (36)$$

where

$$n(t) = |u(t)|^2 n(t_0) + v(t) \quad (37)$$

is the average photon number of the cavity during time evolution. It shows that if the cavity is initially in a mixed state, it will remain in a mixed state, with different particle number distribution varying in time. In the weak coupling regime,  $u(t)$  approaches to zero so that the reduced density matrix at steady limit will reach to the same state as in the other two cases. In the strong coupling regime,  $u(t)$  may not decay to zero so that the average photon number in cavity  $n(t)$  can be very different from the weak coupling regime, namely very different from the BM limit, even though both the exact reduced density matrix and its BM limit are mixed states.

In the next section, we will numerically demonstrate these dynamics for some specifically given spectral density  $J(\omega)$ .

## IV. NUMERICAL ANALYSIS OF THE EXACT NON-MARKOVIAN DYNAMICS

To explicitly see the non-Markovian memory effect in the cavity system when it strongly couples with a reservoir, we consider a general spectral density of the bosonic environment

$$J(\omega) = 2\pi\eta\omega \left( \frac{\omega}{\omega_c} \right)^{s-1} e^{-\omega/\omega_c}, \quad (38)$$

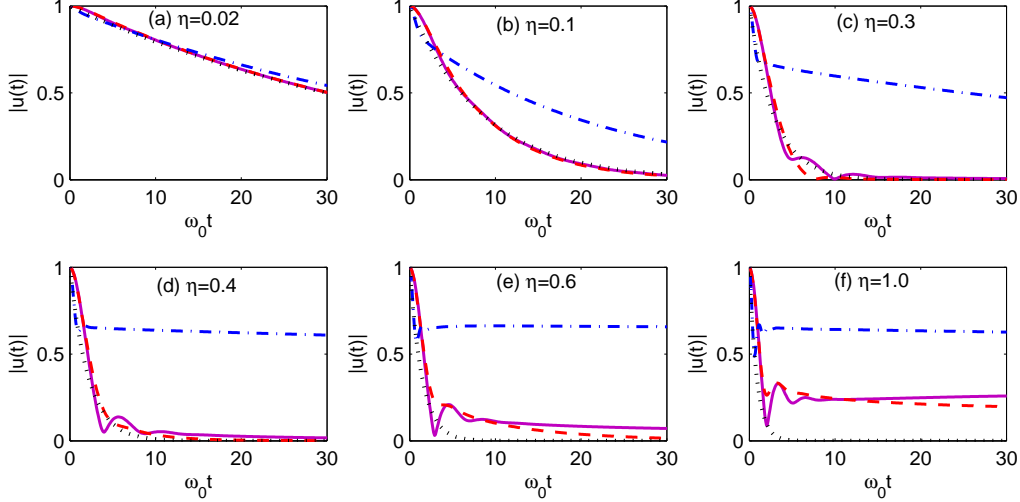


FIG. 1: Comparison of the exact solution of  $|u(t)|$  for sub-Ohmic (solid Magenta line), Ohmic (dashed red line) and super-Ohmic (dash-dotted blue line) with the corresponding BM limit (dotted black line) in the weak coupling ( $\eta \leq 0.3$ ) and the strong coupling ( $\eta \geq 0.4$ ) regimes. Here we have taken the parameters  $\omega_0 = 21.5\text{GHz}$ ,  $\omega_c = \omega_0$ .

where  $\eta$  is the dimensionless coupling strength between the system and reservoir,  $\omega_c$  is the cutoff frequency of the spectrum. The parameter  $s$  classifies the environment as sub-Ohmic ( $0 < s < 1$ ), Ohmic ( $s = 1$ ), and super-Ohmic ( $s > 1$ ). In the following, we set the value of  $s$  to  $1/2$ ,  $1$ , and  $3$  respectively.

For a single-mode cavity with the frequency in the optical regime, it is well known that the BM approximation works very well for a usual thermal environment. However, for a structured reservoir, such as the CROW, the coupling strength between the cavity and reservoir can be controlled by changing the geometrical parameters of the defect cavity and the distance between the cavity and the CROW [15]. Then the BM approximation or the perturbative approximation must be reexamined. Furthermore, when the cavity frequency lies in the microwave regime, the temperature of the environment also becomes non-negligible. Specifically we take the cavity frequency to be  $\omega_0 = 21.5\text{ GHz}$ , i.e.,  $\hbar\omega_0 = 13.83\mu\text{eV}$ , and the temperature is taken at  $T = 2\text{K}$  so that  $k_B T = 172.3\mu\text{eV} \approx 12.5\hbar\omega_0$  [26]. With these experimental parameters as input, we numerically calculate the exact dissipative dynamics of the cavity for three different spectral densities with different coupling strength  $\eta$  in Eq. (38). In addition, the cutoff frequency  $\omega_c$  in Eq. (38) is taken roughly the same order as the cavity frequency  $\omega_0$ , i.e.  $\omega_c \approx \omega_0$ . The detailed numerical results are plotted in Figs. 1-6. Note that the BM limits are the same for three different spectral densities when we take the cut-off frequency  $\omega_c = \omega_0$  since  $u_{\text{BM}}(t)$ ,  $v_{\text{BM}}(t)$  and  $n_{\text{BM}}(t)$  only depends on  $\kappa = J(\omega_0)/2 = \pi\eta\omega_0 e^{-1}$  for all three different spectral densities [see Eq. (38)]. From Eqs. (19), (21) and (22), we can analytically know that  $u(t)$ ,  $v(t)$  and  $n(t)$  in the

BM limit are monotonous change in time as the increases of the coupling  $\eta$ . However, in the exact cases, we will show that the results have qualitative changes from the weak to strong coupling regimes.

In Fig. 1, we show the exact solutions for the absolute value of  $u(t)$  (i.e. the amplitude of the propagating function  $u(t)$  which characterizes the time evolution of the amplitude of the cavity field through the relation  $\langle a(t) \rangle = u(t)\langle a(t_0) \rangle$ ) for Ohmic, sub-Ohmic and super-Ohmic spectral densities in the weak and strong coupling cases with a comparison to the BM limit. For a given coupling strength  $\eta$ , comparing the behavior of the exact  $u(t)$  of different spectral densities with its BM limit, we see that in the very weak coupling limit ( $\eta = 0.02$ ), the difference between the exact amplitude of  $u(t)$  and the BM limit is very small for all the three spectral densities. With the increasing of  $\eta$ , the difference becomes more and more visible. The large difference between the exact result and the BM limit for a strong coupling is a significant manifestation of the non-Markovian memory effect.

For a given spectral density, comparing the behavior of  $|u(t)|$  among different coupling strengths, we find that roughly for  $\eta < 0.3$ ,  $|u(t)|$  decays almost monotonously for all the three spectral densities, except for a short-time oscillation in the beginning. In general, a weak coupling to the reservoir always induces an amplitude damping to the cavity field. However, when  $\eta > 0.3$ , besides a short-time oscillation and decay,  $|u(t)|$  may approach to a nonzero stationary value. In other words, in the strong coupling regime, the non-Markovian memory effect can result in a long-time qualitative change to the time evolution of the cavity field, namely it changes the cavity

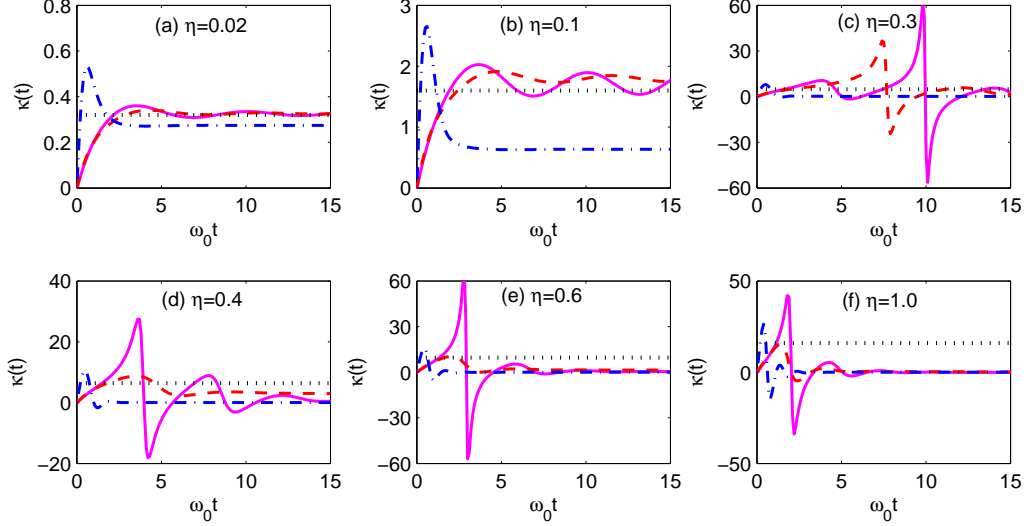


FIG. 2: Comparison of the exact solution of  $\kappa(t)$  for sub-Ohmic (solid Magenta line), Ohmic (dashed red line) and super-Ohmic (dash-dotted blue line) with the corresponding BM limit (dotted black line) in the weak coupling ( $\eta \leq 0.3$ ) and the strong coupling ( $\eta \geq 0.4$ ) regimes. Here we have taken the parameters  $\omega_0 = 21.5\text{GHz}$ ,  $\omega_c = \omega_0$ .

field from a pure damping process in the weak coupling regime to a dissipationless process in the strong coupling regime. In particular, this qualitative change in the exact solution of  $u(t)$  becomes the most significant for the super-Ohmic case, and then for the sub-Ohmic and the Ohmic cases, in comparison with the BM limit, as shown in Fig. 1.

In order to see cleaner the short- and long-time behaviors of  $u(t)$ , we plot  $\kappa(t) = -\text{Re}[\dot{u}(t)/u(t)]$  in Fig. 2 which is the decay coefficient in the master equation (2) that describes the energy dissipation of the cavity. The short-time rapidly increasing of  $\kappa(t)$  in the beginning indicates a fast decay of the cavity field, in agreement with the results shown in Fig. 1. However, with increasing the coupling  $\eta$ , in particular for  $\eta \geq 0.3$ , we see that the decay coefficient  $\kappa(t)$  shows a very different time-dependence. It has a large oscillation between an equal positive and negative bounded value and then approaches to zero. The oscillation indicate that the cavity dissipates energy into the reservoir first and gains the energy back from the reservoir. Then the zero steady value indicates that after the oscillation, the cavity becomes dissipationless. This is why  $|u(t)|$  has a nonzero steady value, as we seen in Fig. 1.  $|u(t)|$  of the super-Ohmic reservoir maintains a largest nonzero steady value in the strong coupling regime because its short-time oscillation occurs in the shortest time within which only the smallest energy is dissipated. While, in the BM limit,  $\kappa$  keeps in a nonzero constant, namely the cavity is always in a dissipation state.

Therefore, we can conclude that the super-Ohmic reservoir contains the strongest non-Markovian memory

effect shown in the long-time dissipationless process for the cavity field, while the sub-Ohmic reservoir involves the strongest short-time oscillating non-Markovian dynamics. In the previous investigations on non-Markovian effect, one has observed the short-time oscillation behavior in the dissipation processes in the weak coupling regime [8]. The long-time dissipationless stationary behavior in the strong coupling regime may only be revealed from a nonperturbation theory, as we shown here.

To see clearly the significance of the temperature effect of the non-Markovian memory dynamics, we show in Fig. 3 the time evolution of the correlation function  $v(t)$  in the weak and strong coupling regimes and compare the results with the BM solution again. The correlation function  $v(t)$  characterizes the temperature-dependent noise effect (the average photon correlation through the reservoir). Meanwhile,  $v(t)$  is also the main contribution to the average photon number inside the cavity induced mainly by thermal noise dynamics, see Eq. (18). It is shown in Table I that, in the zero-temperature limit  $T \rightarrow 0$ ,  $v(t) = 0$ . However, at a finite temperature, it will behave very differently for different couplings.

In the very weak coupling limit ( $\eta = 0.02$ ), the exact  $v(t)$  is almost the same as the BM solution, except for the sub-Ohmic spectral density in which  $v(t)$  shows a long-time oscillation, as a weak non-Markovian memory effect. With the increasing of  $\eta$ , the quantitative difference between the exact solution and its BM limit is enlarged for all the three spectral densities. Comparing the behavior of  $v(t)$  among different  $\eta$  for a given spectral density, we find that, similar to the behavior of  $|u(t)|$ , a short-time oscillation (almost invisible in Fig. 3) of  $v(t)$  exists for



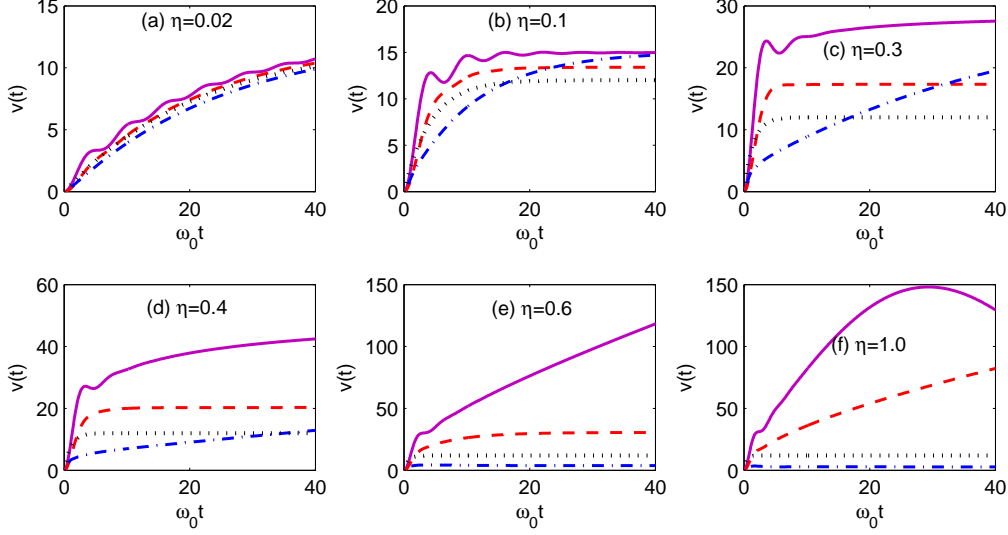


FIG. 3: Exact solution of  $v(t)$  for sub-Ohmic (solid Magenta line), Ohmic (dashed red line) and super-Ohmic (dash-dotted blue line) with the corresponding BM limit (dotted black line) in the weak coupling ( $\eta \leq 0.3$ ) and the strong coupling ( $\eta \geq 0.4$ ) regimes. Here  $\omega_0 = 21.5\text{GHz}$ ,  $\omega_c = \omega_0$ , and  $T = 2\text{K}$ .

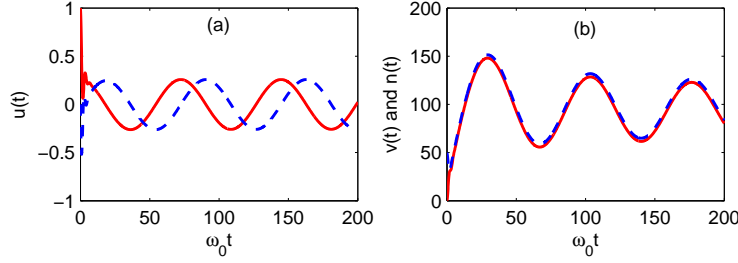


FIG. 4: Long time behavior of  $u(t)$ ,  $v(t)$  and  $n(t)$  for sub-Ohmic spectral density with  $\eta = 0.1$ . Figure (a) shows  $\text{Re}[u(t)]$  (red solid line) and  $\text{Im}[u(t)]$  (dashed blue line). Figure (b) shows  $v(t)$  (red solid line) and  $n(t)$  (dashed blue line). Here  $\omega_0 = 21.5\text{GHz}$ ,  $\omega_c = \omega_0$ ,  $T = 2\text{K}$  and  $n(t_0) = 50$ .

all the three spectral densities. In the long-time limit, for different coupling strengths, the stationary values of the exact  $v(t)$  for all three different spectral densities can be very different from the BM result. The numerical result in Fig. 3 clearly shows that  $v(t \rightarrow \infty)$  is far away from the BM limit  $v_{\text{BM}}(t \rightarrow \infty) = \bar{n}(\omega_0, T)$  in the strong coupling regime, as we discussed analytically in the last section.

In particular, continuously increasing the coupling strength  $\eta$  shows that the sub-Ohmic case manifests a significantly qualitative difference from the BM limit. The time oscillation behavior of  $v(t)$  for the sub-Ohmic case is very strong and maintains for a longer time when the coupling  $\eta$  becomes larger. This long-time oscillation behavior comes actually from the long-time periodical oscillation of  $u(t)$ , as we shown in Fig. 4. However, in the strong coupling regime,  $v(t)$  of the super-Ohmic spectral density

increases slower than the BM limit with increasing  $\eta$  but the overall time evolution is qualitatively the same as in the BM limit. In other words, the temperature-induced noise effect becomes very important for sub-Ohmic case and then for the Ohmic case while it may be minor for the super-Ohmic reservoir in the strong coupling regime, which is quite different in comparison with the solution in the weak coupling cases, see Fig. 3.

To show the totally non-Markovian memory effect distributing in the amplitude damping and the noise dynamics, we examine the average photon number inside the cavity which is analytically given by Eq. (18). From equation (18), we see that the average photon number contains two terms, the first term is the decay of the initial average photon number determined by the propagating function  $u(t)$ , and the second term is just the correlation function  $v(t)$  as a noise effect which sensitively

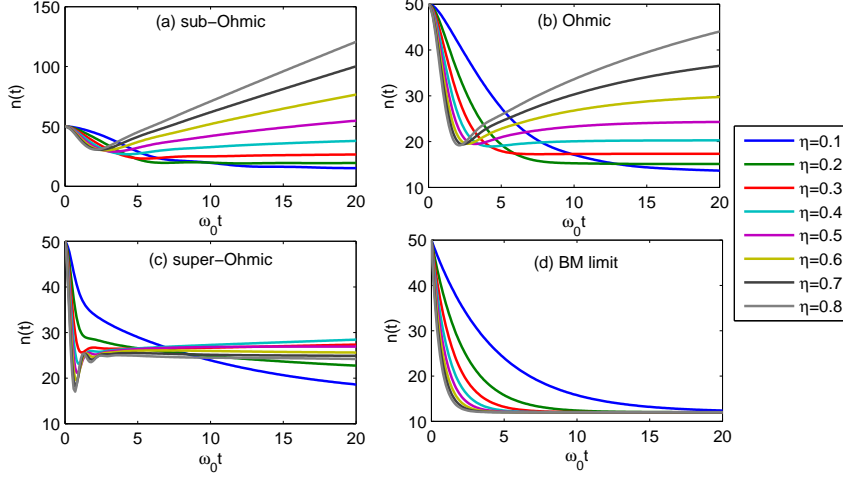


FIG. 5: Time evolution of the average particle number  $n(t)$ . Here  $\omega_0 = 21.5\text{GHz}$ ,  $\omega_c = \omega_0$ ,  $T = 2\text{K}$ , and  $n(t_0) = 50$ .

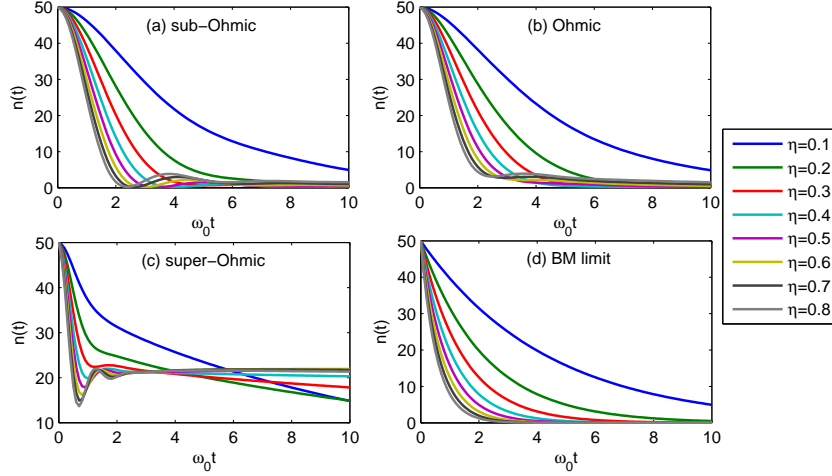


FIG. 6: Time evolution of the average particle number  $n(t)$ . Here  $\omega_0 = 21.5\text{GHz}$ ,  $\omega_c = \omega_0$ ,  $T = 0.002\text{K}$ , and  $n(t_0) = 50$ .

depends on the initial temperature of the reservoir. The numerical solutions based on the exact solution (18) for three different spectral densities are plotted in Fig. 5 with a comparison to the BM result.

In the weak coupling regime (roughly  $\eta < 0.3$ ), as we see the exact solution of  $n(t)$  shows a monotonic decay, similar to that in the BM limit. However, in the strong coupling regime (roughly  $\eta > 0.3$ ), a revival process occurs in the exact  $n(t)$ , namely the average photon number decays faster in the very beginning and then after a short-time oscillation (or no oscillation), it revives till it reaches to a steady value. This behavior does not show up in the BM approximation. In fact, the plots in Fig. 5 show that the time evolution of the average photon number undergoes a critical transition from the weak to strong cou-

pling regimes. The transition regime here corresponds roughly to  $\eta \simeq 0.3$ . This critical transition results in a competition between the non-Markovian memory effect induced dissipationless phenomena with the thermal noise dynamics in the strong coupling in open systems.

Furthermore, this critical transition can still be seen when we reduce the initial temperature of the reservoir to a very low value, see Fig. 6 where  $T = 0.002\text{K}$ . The critical transition is clearly shown up for the super-Ohmic case. For the Ohmic and sub-Ohmic cases, the critical phenomena is not so significant but the average photon number can reach to a very small but nonzero steady value in the strong coupling regime. This result indicates that the initial temperature of the reservoir may serve as a sensitive control parameter to control the co-

herence photon number in cavity dynamics in the very low temperature regime, as we have also discussed in the analytical solution presented in the last section.

Put all these analysis together, we find that the non-Markovian memory effect can qualitatively change the dissipation dynamics of the cavity field in the strong coupling regime, in particular for the super-Ohmic reservoir. Meantime, the non-Markovian memory effects play an significant role to the thermal noise dynamics, in particular for the sub-Ohmic reservoir. These interesting phenomena worth further investigation in other open systems.

## V. CONCLUSION

In summary, we have solved analytically the exact master equation, and obtained the general expression for the reduced density operator of the cavity as well as the general solution of the mean model amplitude and the average photon numbers in the cavity. We take three different initial states: the vacuum state, the coherent state and the mixed state, to show the different non-Markovian time evolution of the cavity state. We find that (i) The solution of the exact master equation is very different from the BM approximation due to the non-Markovian memory effect. In the exact non-Markovian case, different initial states may result in different evolution states and different steady states. While the BM solution at steady state limit are the same, independent of the initial states. (ii) For the exact non-Markovian evo-

lution process, temperature effect can play an important role, and it drastically changes the thermal noise dynamics. (iii) The decoherence of cavity coherent field arises from both the reservoir-induced damping effect (energy dissipation) and the temperature-dependent noise effect. Both of them can be controlled by varying the coupling between the cavity and the reservoir and lowering the temperature of the reservoir, as shown in our exact solution. These exact analytical cavity dynamics show the extreme importance of the non-Markovian effect and are numerically demonstrated. Moreover, the numerical results show that, the non-Markovian memory effect qualitatively changes the amplitude damping behavior of the cavity field as well as the thermal noise dynamics from the weak coupling regime to strong coupling regime, which does not occur in the BM limit. In particular, we show that the non-Markovian memory effect leads to a long-time dissipationless process to the cavity field in the strong coupling regime, and meantime it results in a critical transition dynamics when we vary the cavity system from the weak to strong couplings with the reservoir. Further investigations of all these phenomena in other open systems are in progress.

*Acknowledgements:* We thank Chan U Lei for some numerical checks. This work is supported by the National Science Council of ROC under Contract No. NSC-96-2112-M-006-011-MY3, NSFC with grant No.10874151, 10935010, NFRPC with grant No. 2006CB921205; Program for New Century Excellent Talents in University (NCET), and Science Foundation of Chinese University.

- 
- [1] H. J. Carmichael, *An Open Systems Approach to Quantum Optics*, Lecture Notes in Physics, Vol. m18 (Springer-Verlag, Berlin, 1993).
  - [2] M. J. Hartmann, F. G. S. Brandao, and M. Plenio, *Nature Phys.* **2**, 849 (2006).
  - [3] T. Pellizzari, *Phys. Rev. Lett.* **79**, 5242 (1997).
  - [4] A. Biswas and D. A. Lidar, *Phys. Rev. A* **74**, 062303 (2006).
  - [5] Q. A. Turchette, C. J. Myatt, B. E. King, C. A. Sackett, D. Kielpinski, W. M. Itano, C. Monroe, and D. J. Wineland, *Phys. Rev. A* **62**, 053807 (2000); C. J. Myatt et al., *Nature (London)* **403**, 269 (2000).
  - [6] S. Maniscalco, J. Piilo, F. Intravaia, F. Petruccione, and A. Messina, *Phys. Rev. A* **69**, 052101 (2004).
  - [7] J. Piilo and S. Maniscalco, *Phys. Rev. A* **74**, 032303 (2006).
  - [8] J. Paavola, J. Piilo, K. -A. Suominen, and S. Maniscalco, *Phys. Rev. A* **79**, 052120 (2009).
  - [9] N. Stefanou and A. Modinos, *Phys. Rev. B* **57**, 12127 (1998).
  - [10] A. Yariv, Y. Xu, R. K. Lee, and A. Scherer, *Opt. Lett.* **24**, 711 (1999).
  - [11] M. Bayindir, B. Temelkuran, and E. Ozbay, *Phys. Rev. Lett.* **84**, 2140 (2000).
  - [12] Y. Xu, Y. Li, R. K. Lee, and A. Yariv, *Phys. Rev. E* **62**, 7389 (2000).
  - [13] S. Olivier, C. Smith, M. Rattier, H. Benisty, C. Weisbuch, T. Krauss, R. Houdre, and U. Oesterle, *Opt. Lett.* **26**, 1019 (2001).
  - [14] L. L. Lin, Z. Y. Li, and B. Lin, *Phys. Rev. B* **72**, 165330 (2005).
  - [15] Y. Liu, Z. Wang, M. Han, S. Fan, and R. Dutton, *Opt. Express* **13**, 4539 (2005).
  - [16] S. Longhi, *Phys. Rev. A* **74**, 063826 (2006).
  - [17] M. A. Nielsen and I. L. Chuang, *Quantum Computation and Quantum Information*, (Cambridge University Press, 2000).
  - [18] H. P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems*, (Oxford University Press, New York, 2007).
  - [19] U. Weiss, *Quantum Dissipative Systems*, (3rd Ed. World Scientific, Singapore, 2008).
  - [20] B. L. Hu, J. P. Paz, and Y. H. Zhang, *Phys. Rev. D* **45**, 2843 (1992).
  - [21] R. P. Feynman and F. L. Vernon, *Ann. Phys.* **24**, 118 (1963).
  - [22] M. W. Y. Tu and W. M. Zhang, *Phys. Rev. B* **78**, 235311 (2008).
  - [23] M. W. Y. Tu, M. T. Lee and W. M. Zhang, *Quantum Inf. Process* **8**, 631 (2009).

- [24] J. H. Au, and W. M. Zhang, Phys. Rev. A, **76**, 042127 (2007).
- [25] J. H. Au, M. Feng and W. M. Zhang, Quant. Info. Comput, **9**, 0317, (2009).
- [26] D. Meschede and H. Walther, Phys. Rev. Lett. **54**, 551 (1985).
- [27] A. J. Leggett, S. Chakravarty, A. T. Dorsey, M.P. Fisher, A. Garg, and W. Zwerger, Rev. Mod. Phys. **59**, 1 (1987).
- [28] W. M. Zhang, D. H. Feng and R. Gilmore, Rev. Mod. Phys. **62**, 867 (1990).
- [29] L. P. Kadanoff, G. Baym, *Quantum Statistical Mechanics*, (Benjamin, New York, 1962)
- [30] J. S. Jin, M. T. W. Tu, W. M. Zhang and Y. J. Yan, arXiv: 0910.1765.